

Let's Write a Parser

Ionuț G. Stan – I T.A.K.E. – May 2016

About Me

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- Software Developer at [Eloquentix](#)

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- I work mostly with Scala
- I like FP, programming languages, compilers
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- I occasionally blog on [igstan.ro](#)

Plan

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- Vehicle Language: μ ML

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- Compilers Overview

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- Compilers Overview
- Parsing: Intuitions and Live Coding

Vehicle Language: μ ML

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8. Parenthesized expressions: `(a + b)`

Vehicle Language: μ ML

9. Let blocks/expressions:

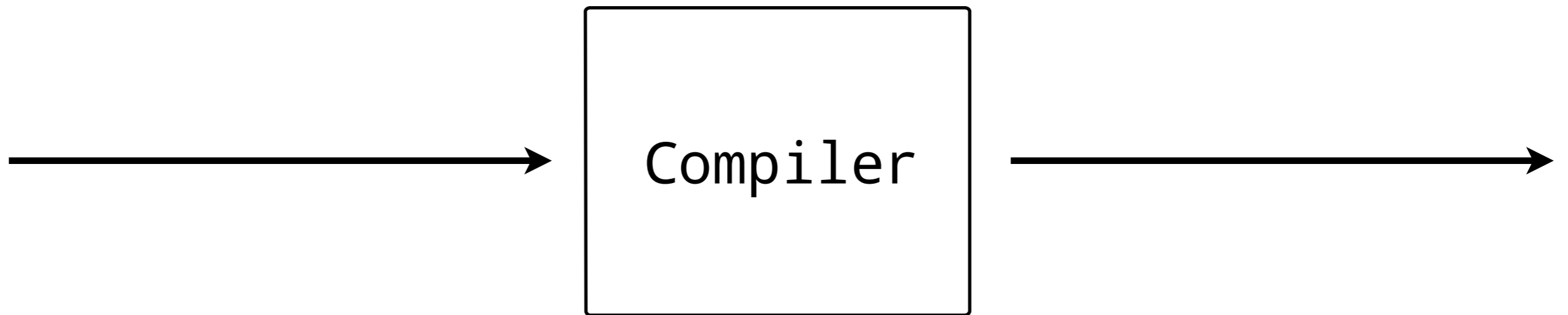
```
let  
  val name = ...  
in  
  name  
end
```

Small Example

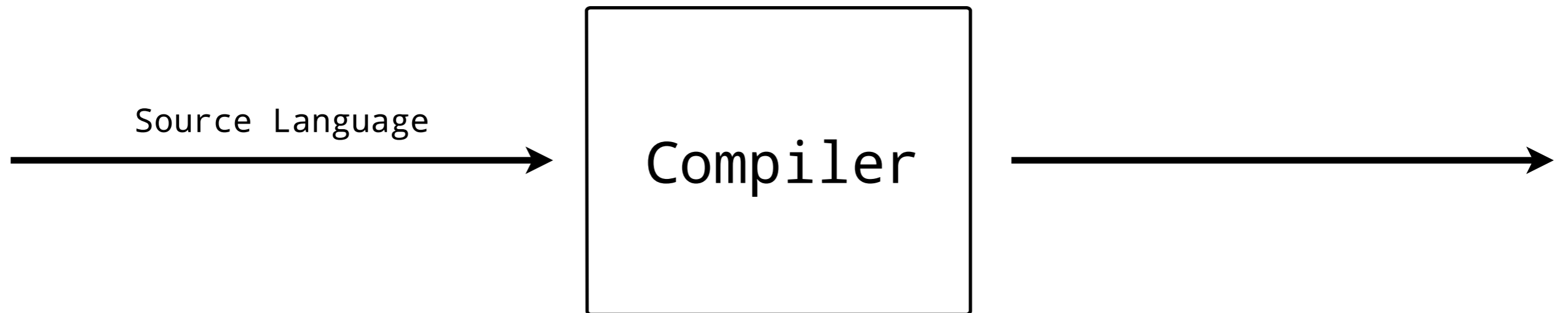
```
let
  val inc =
    fn a => a + 1
in
  inc 42
end
```

Compilers Overview

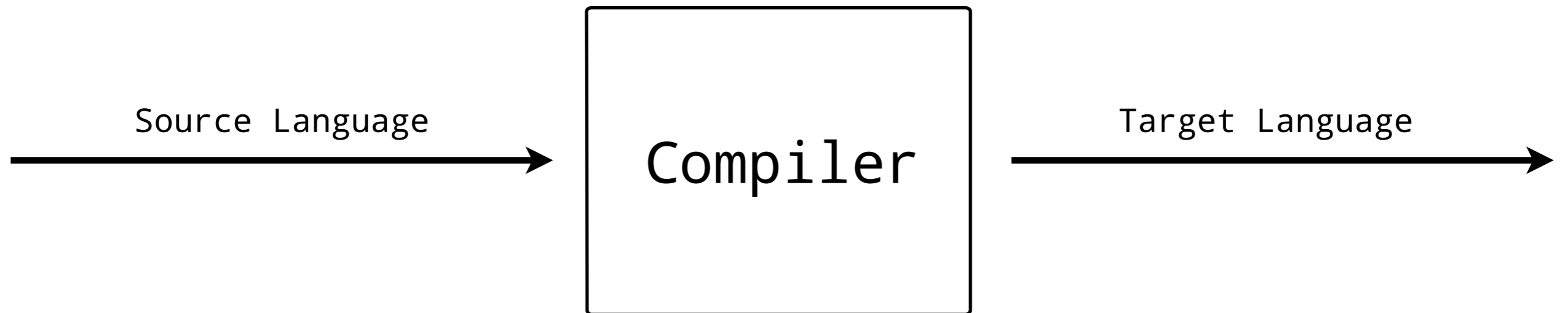
Compilers Overview



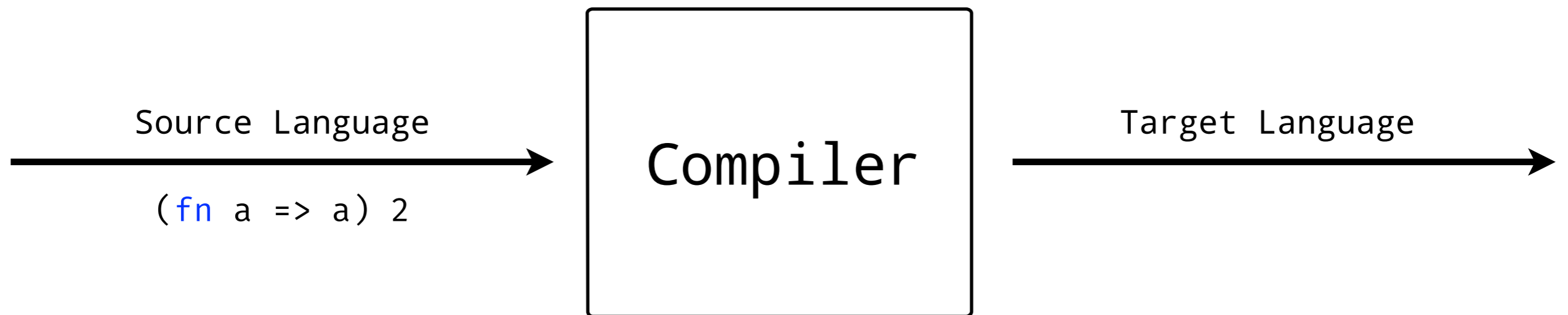
Compilers Overview



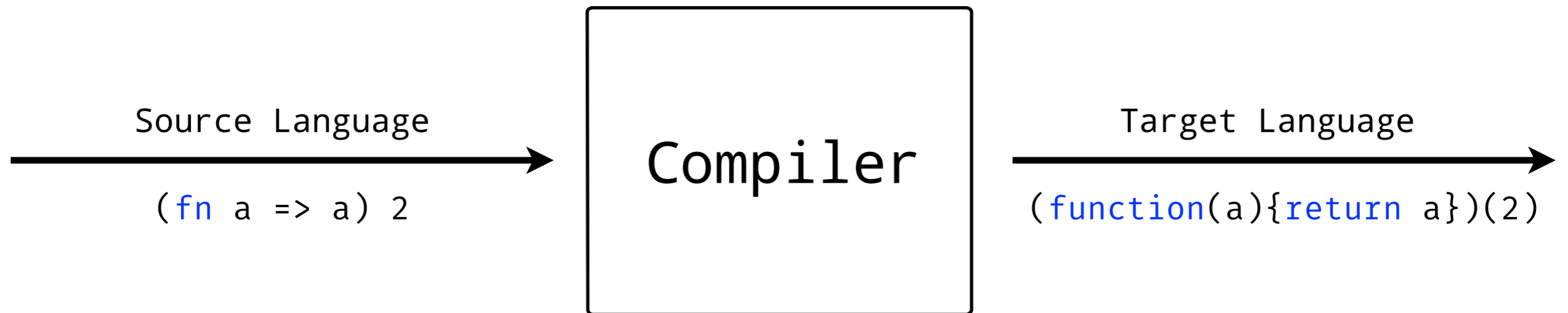
Compilers Overview



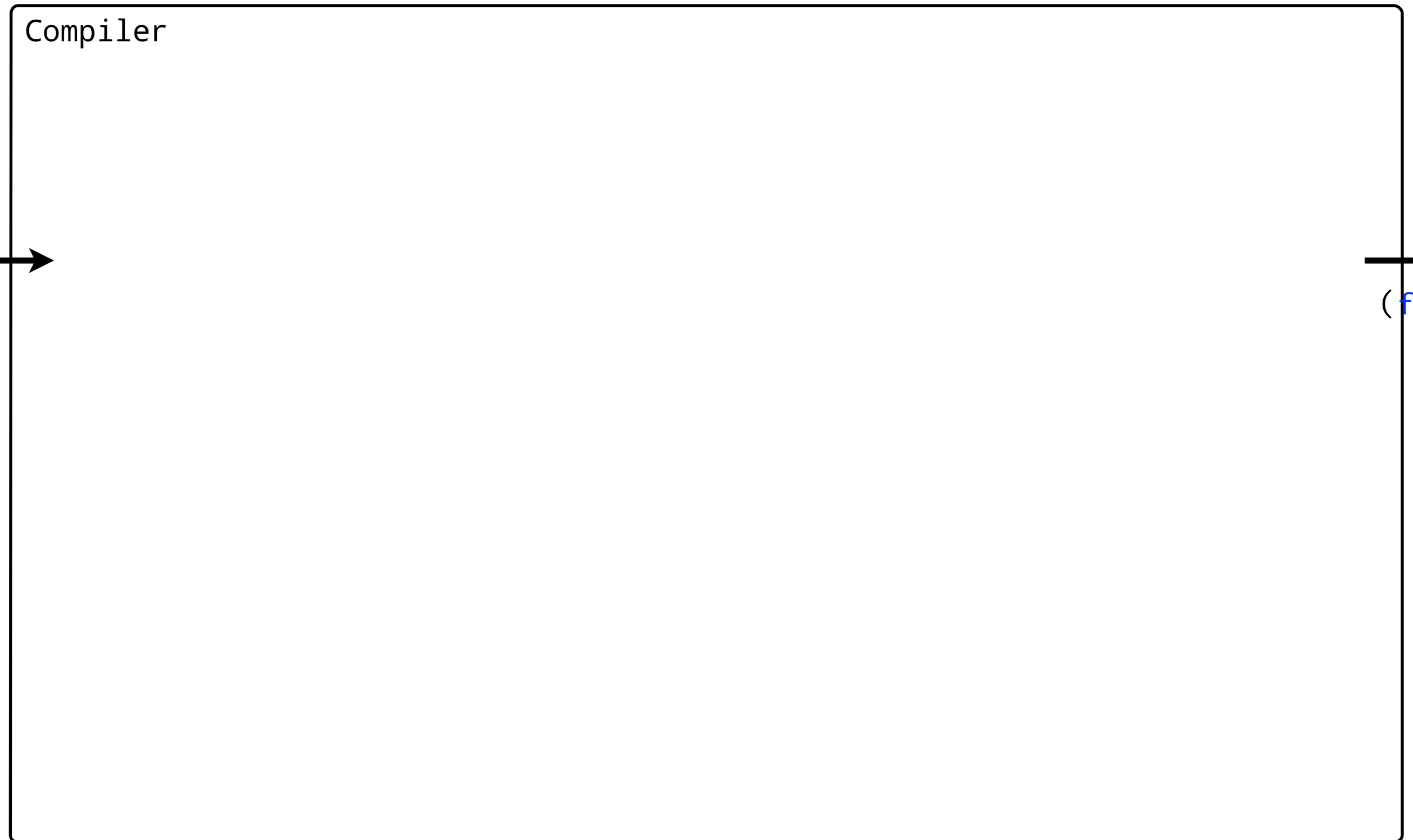
Compilers Overview



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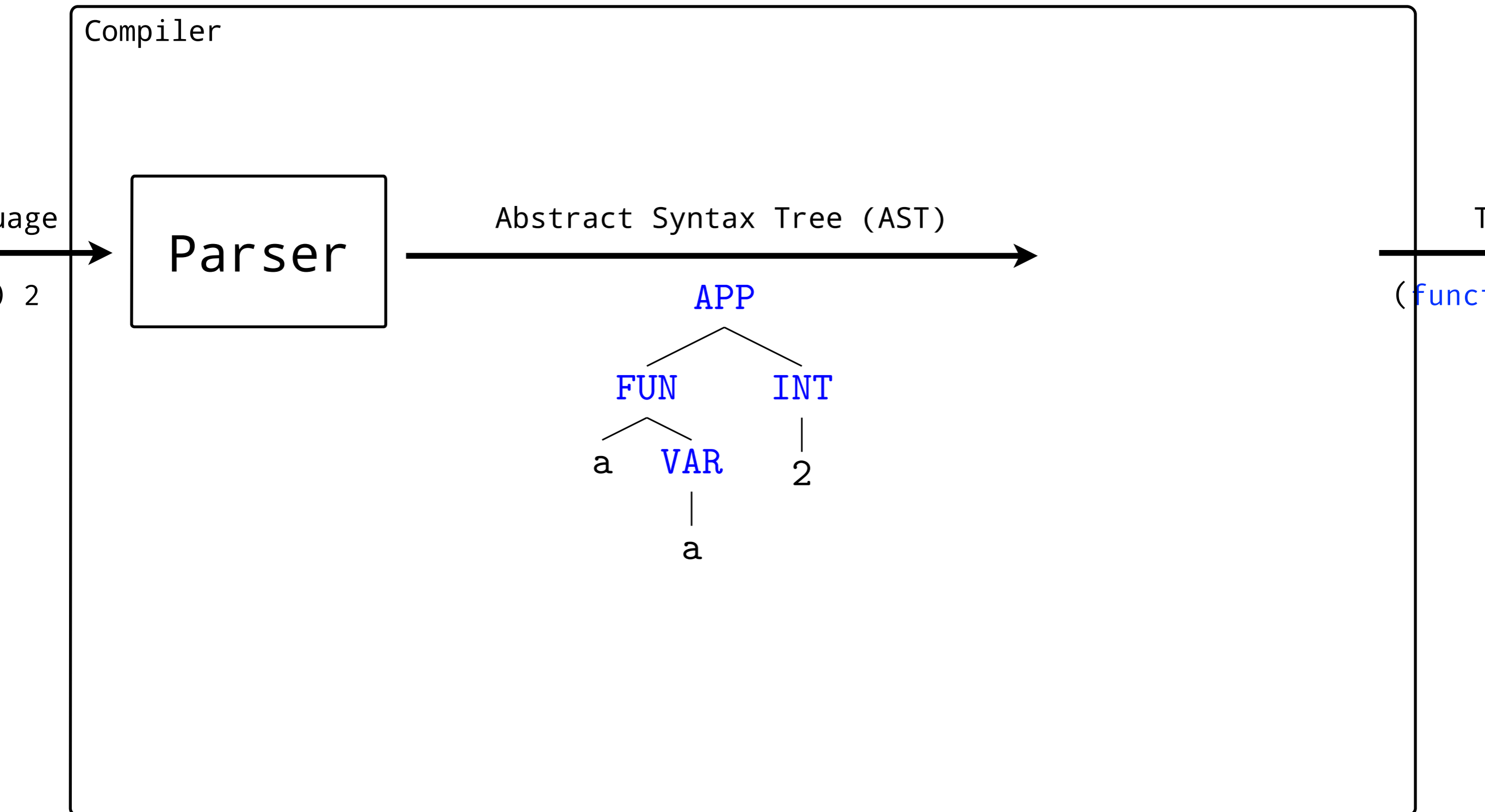
Compilers Overview



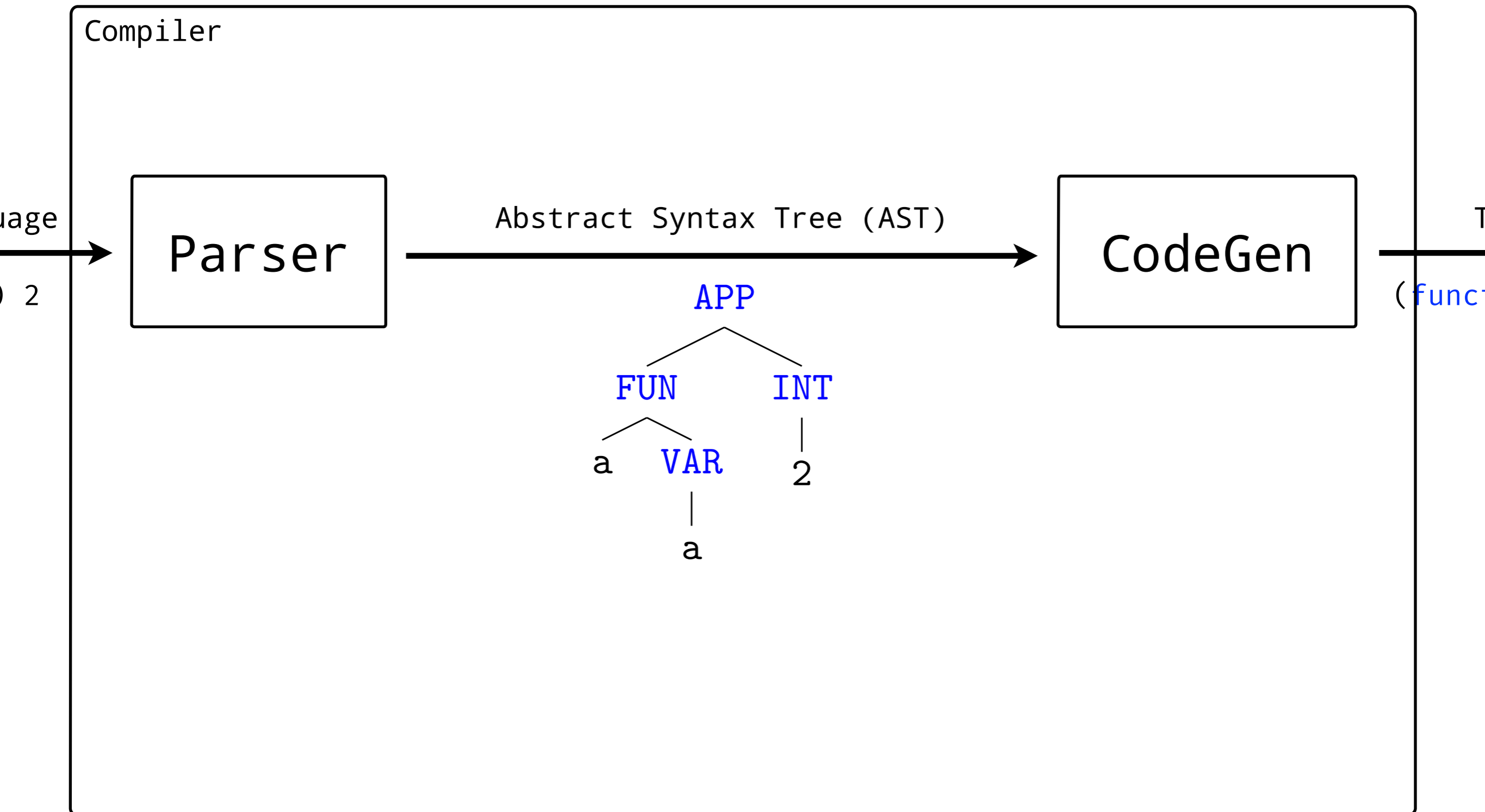
Parsing



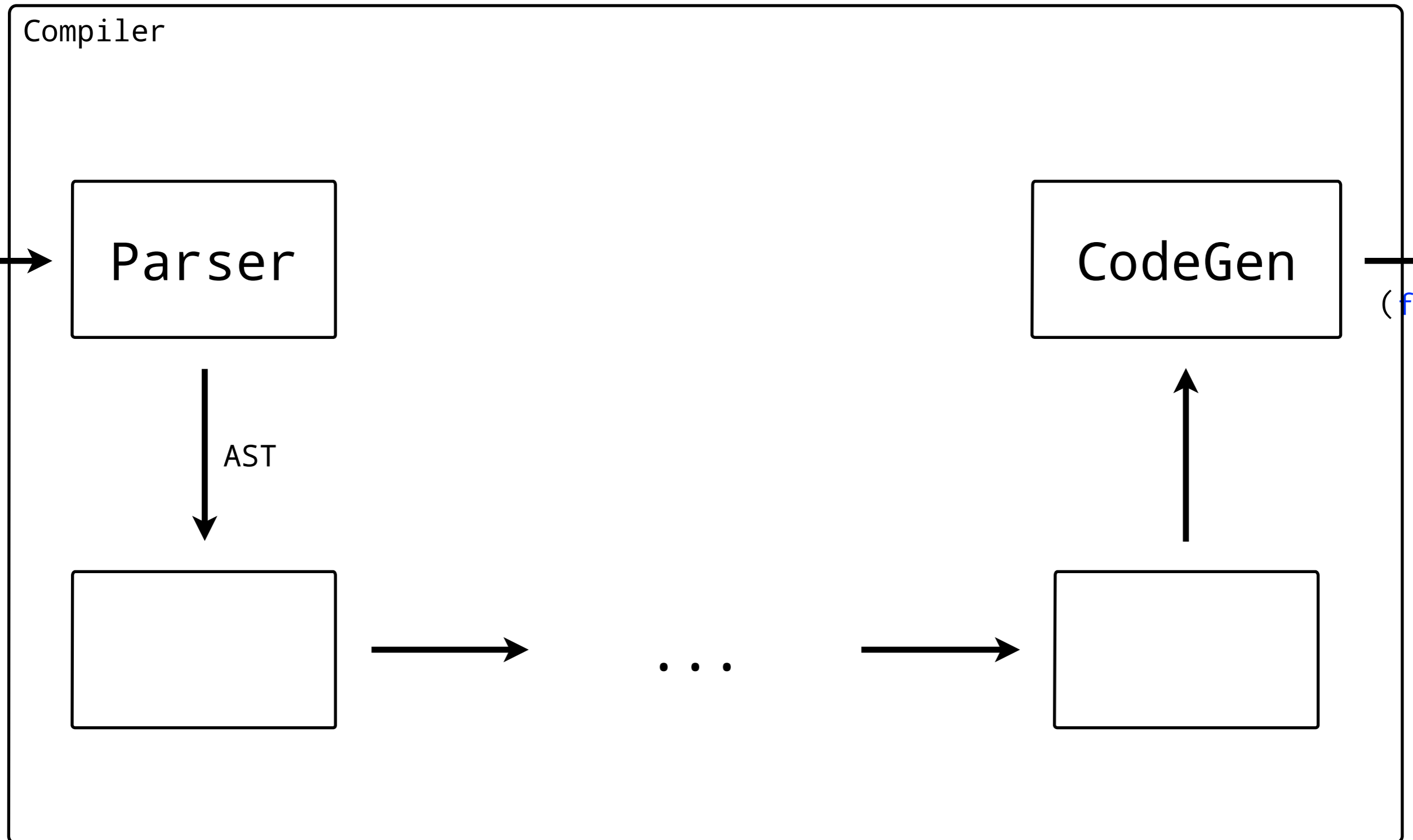
Abstract Syntax Tree



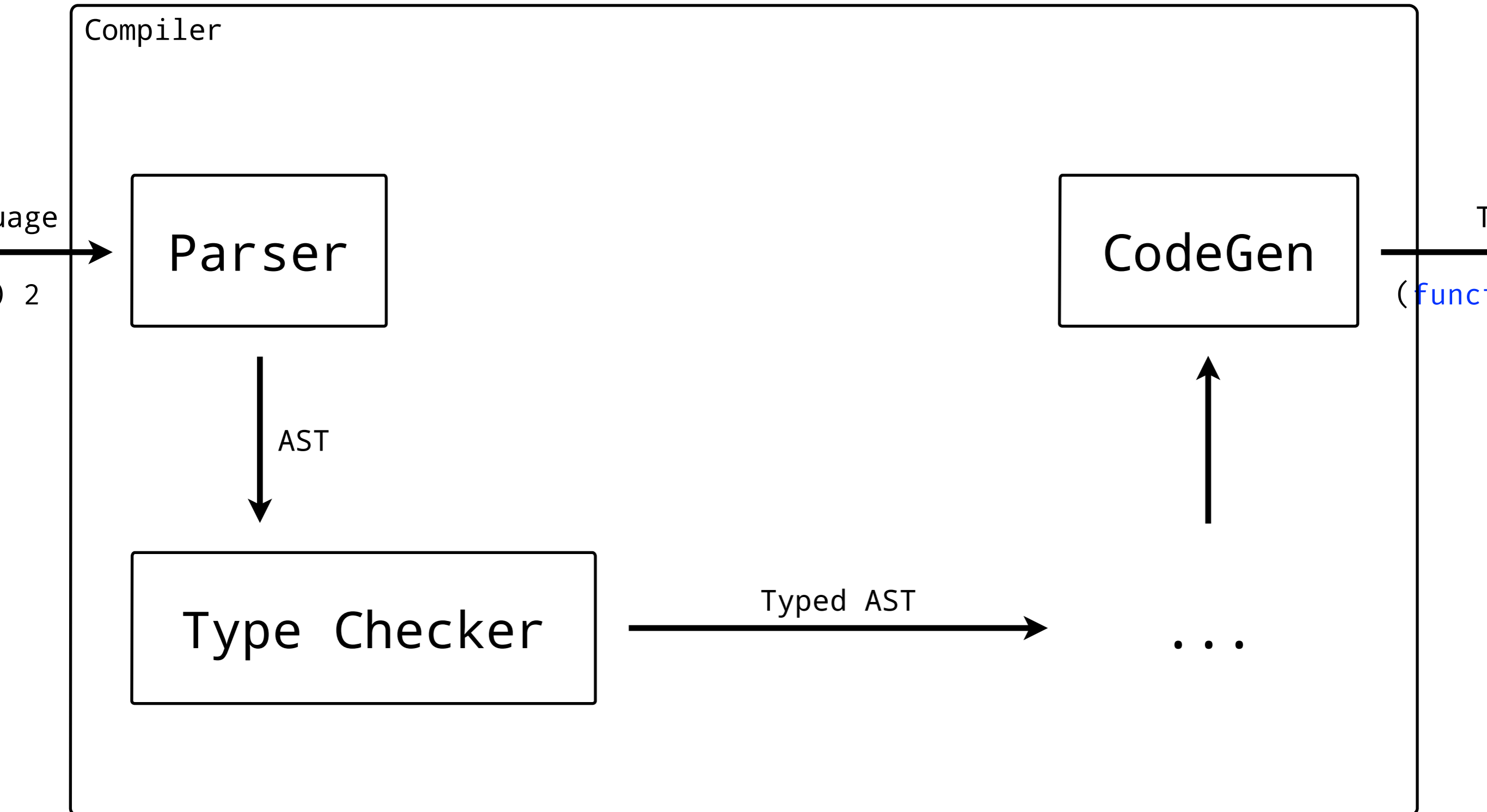
Code Generation



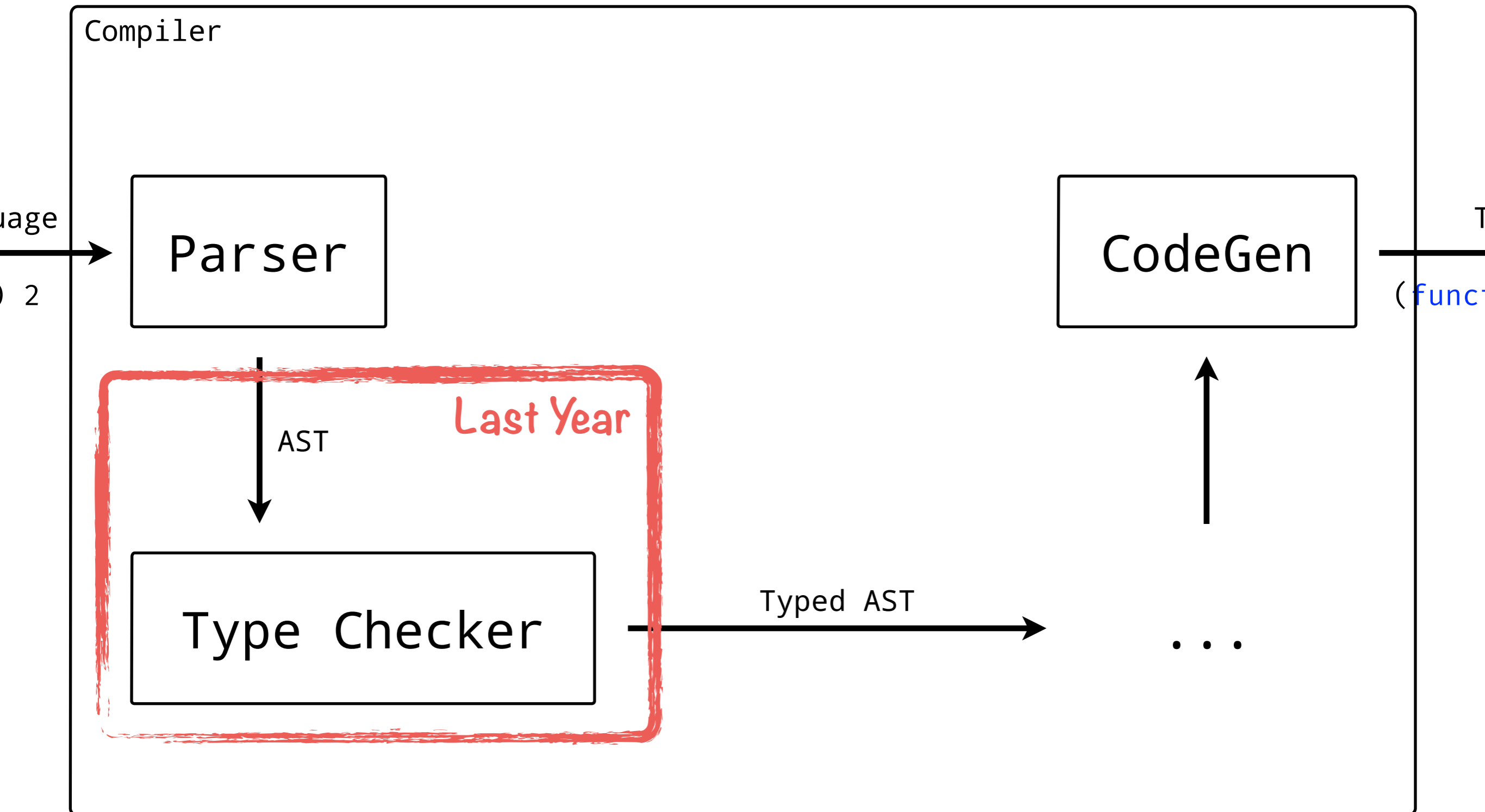
Many Intermediate Phases



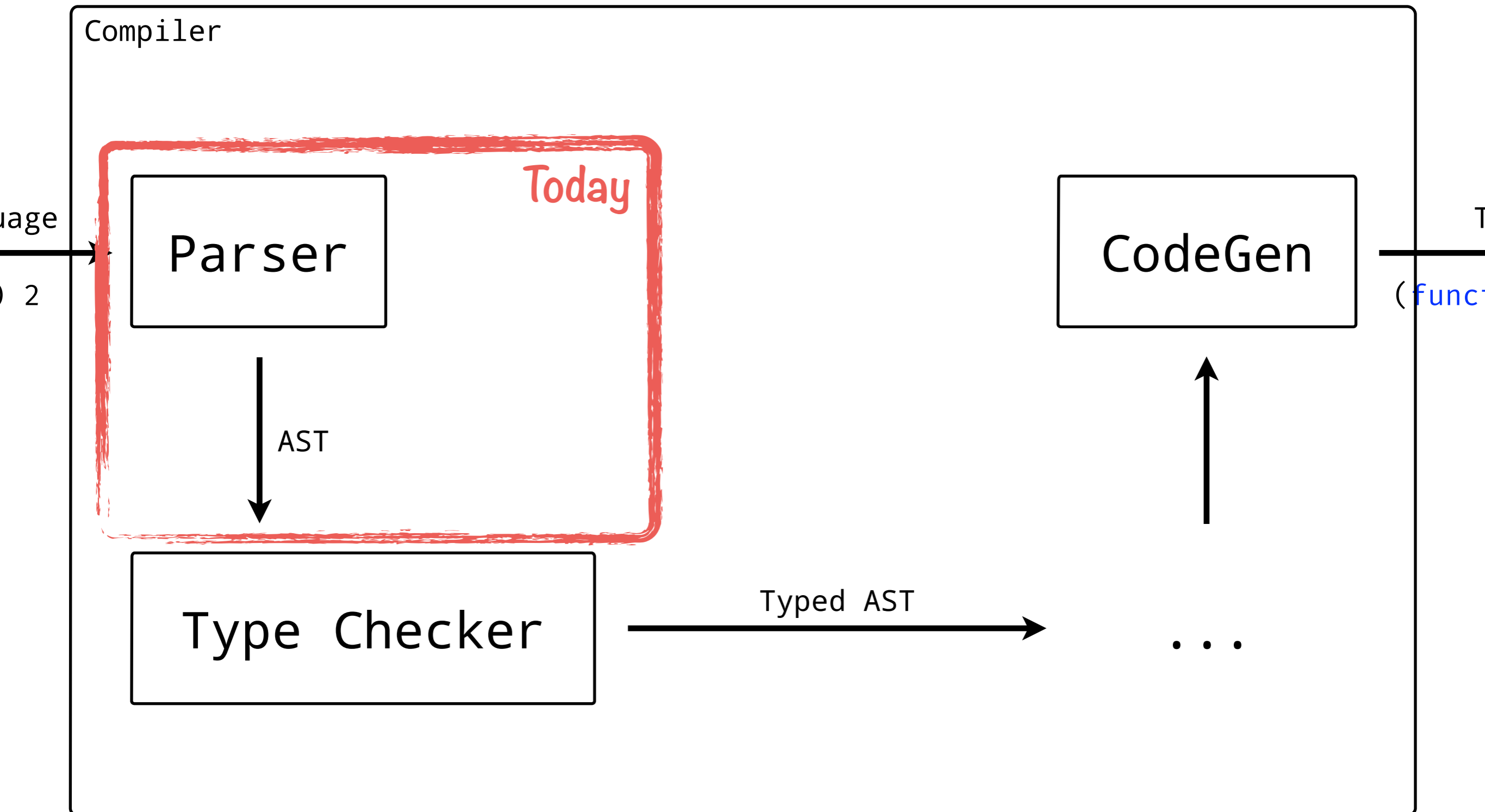
Type Checking



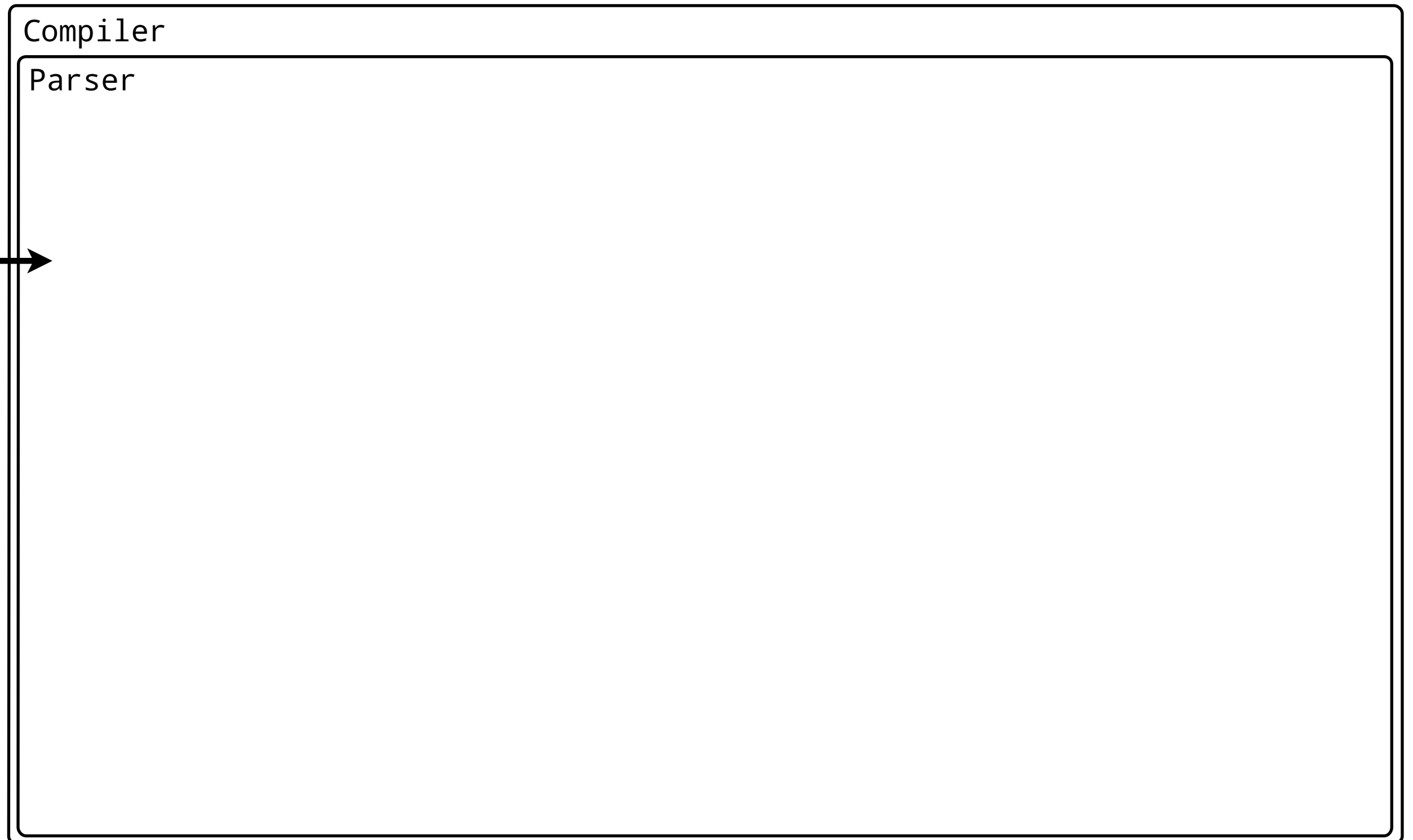
Last Year's Talk



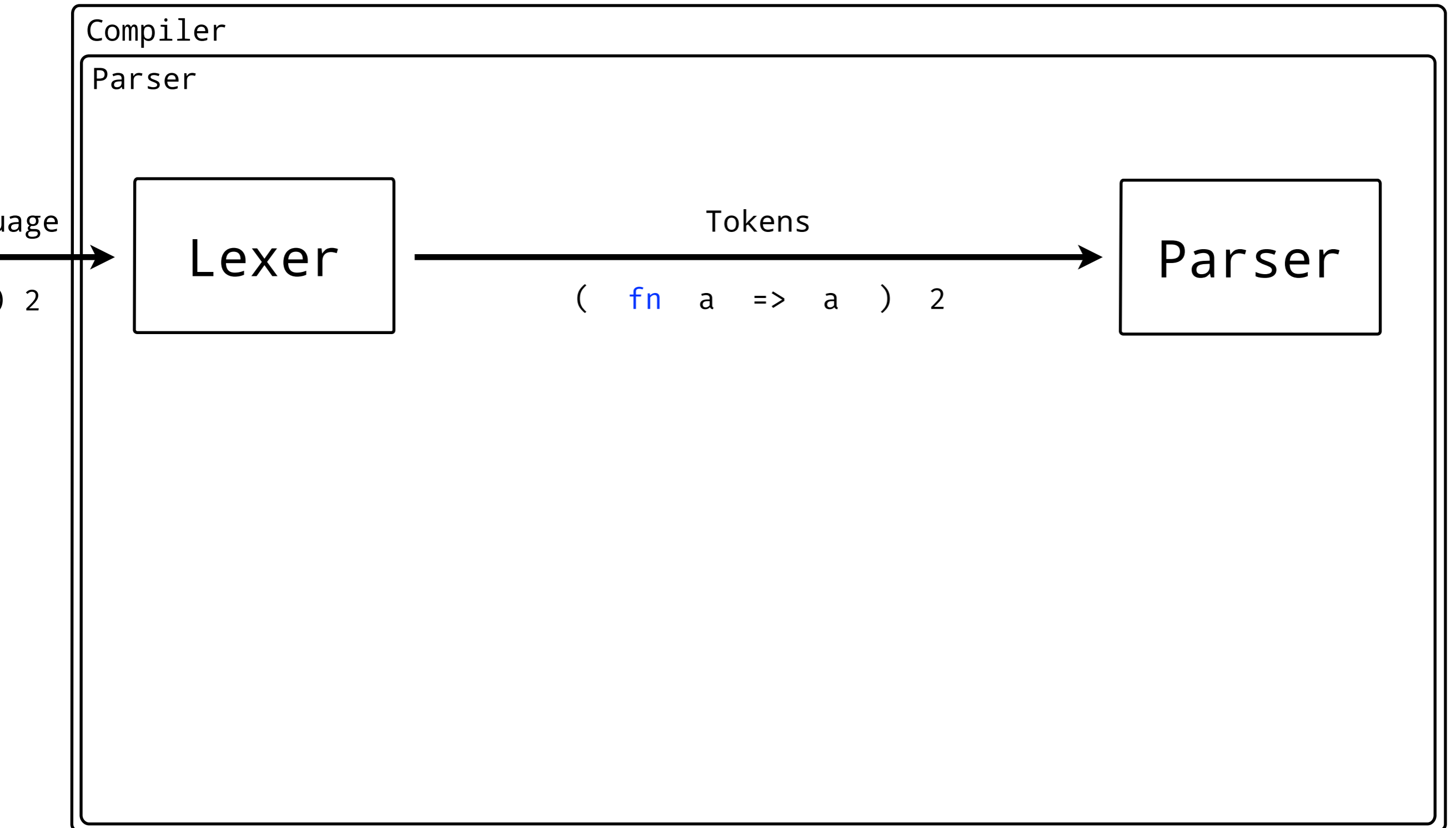
Today's Talk



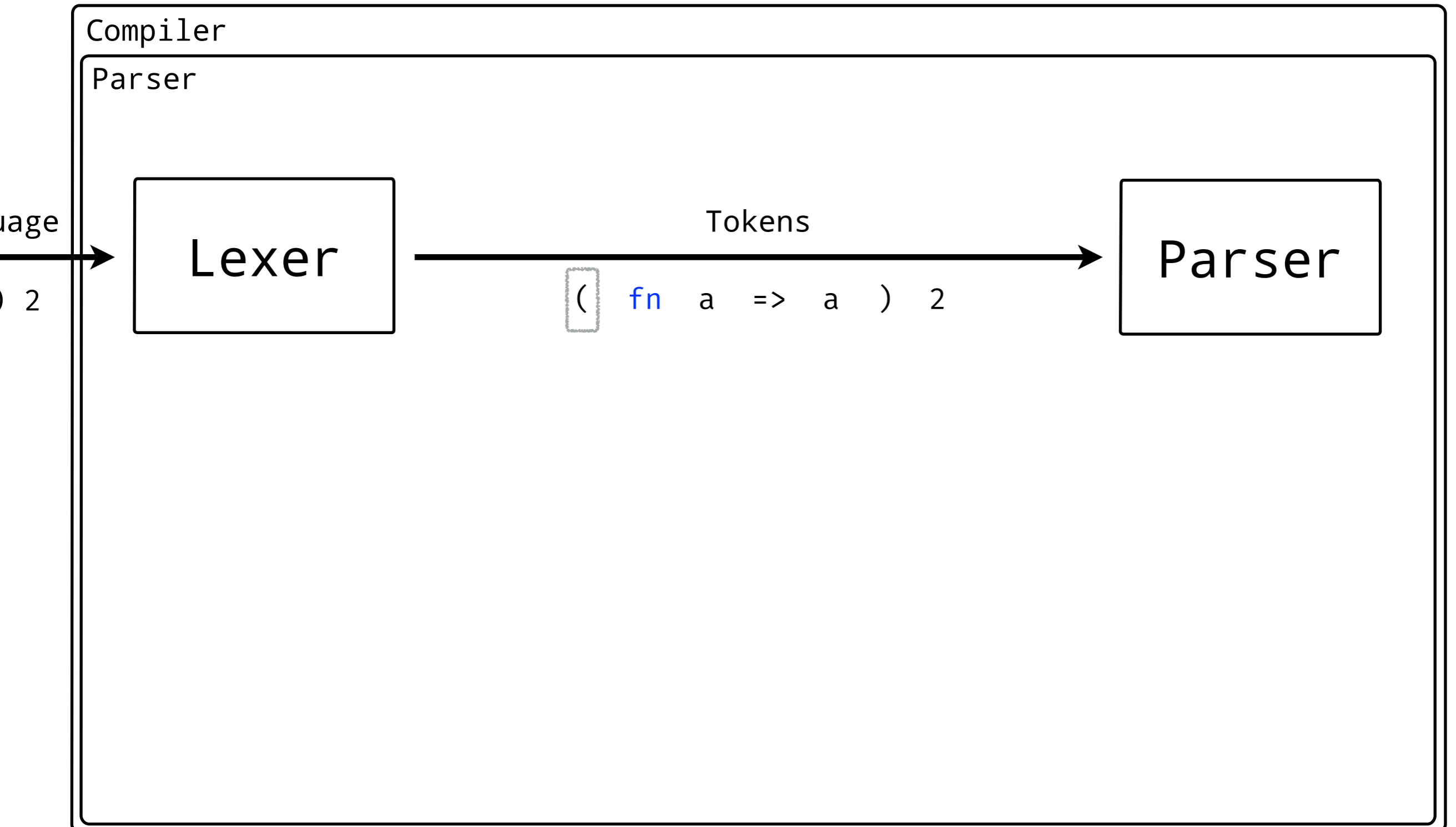
Parsing



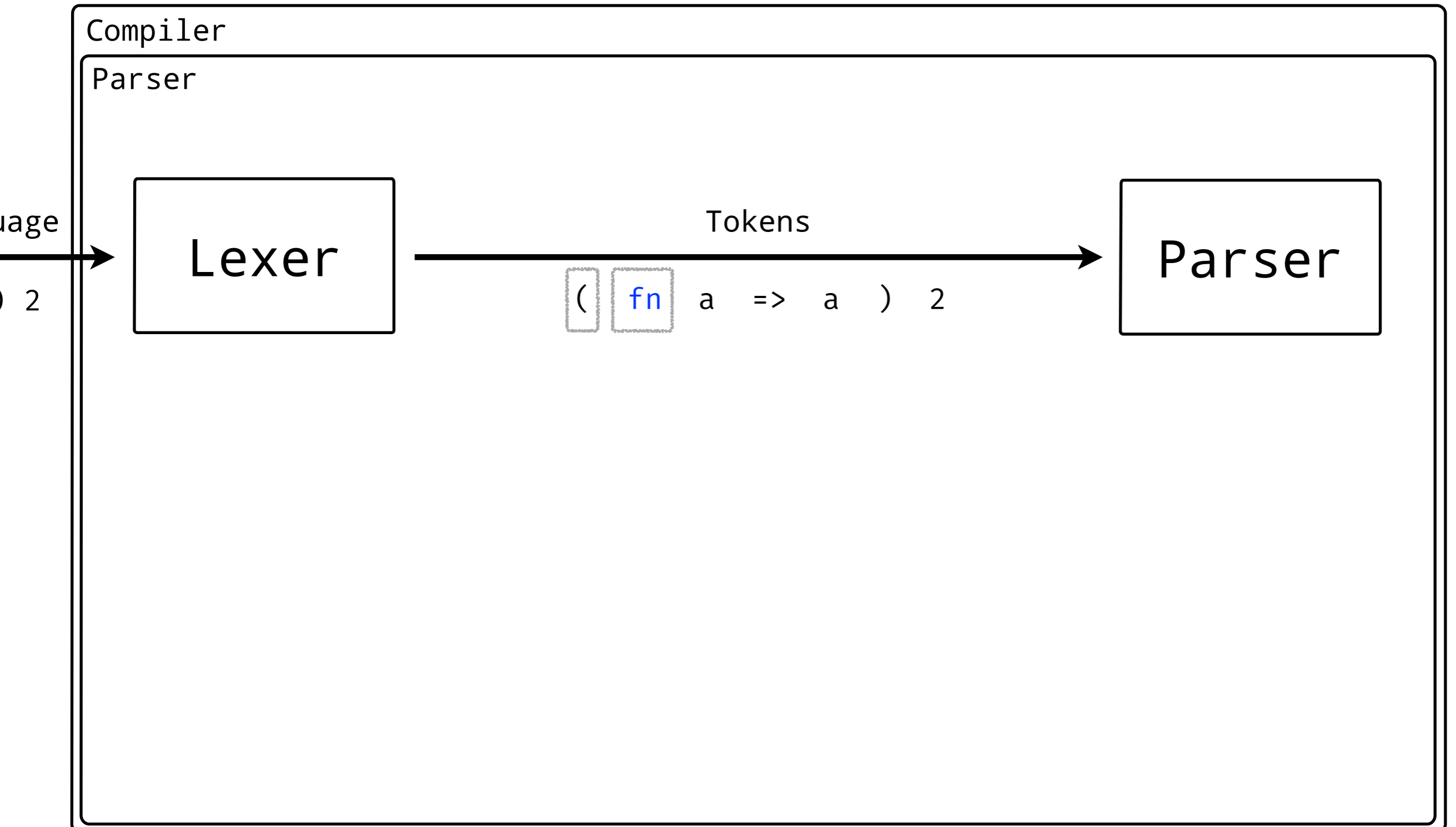
Lexing + Parsing



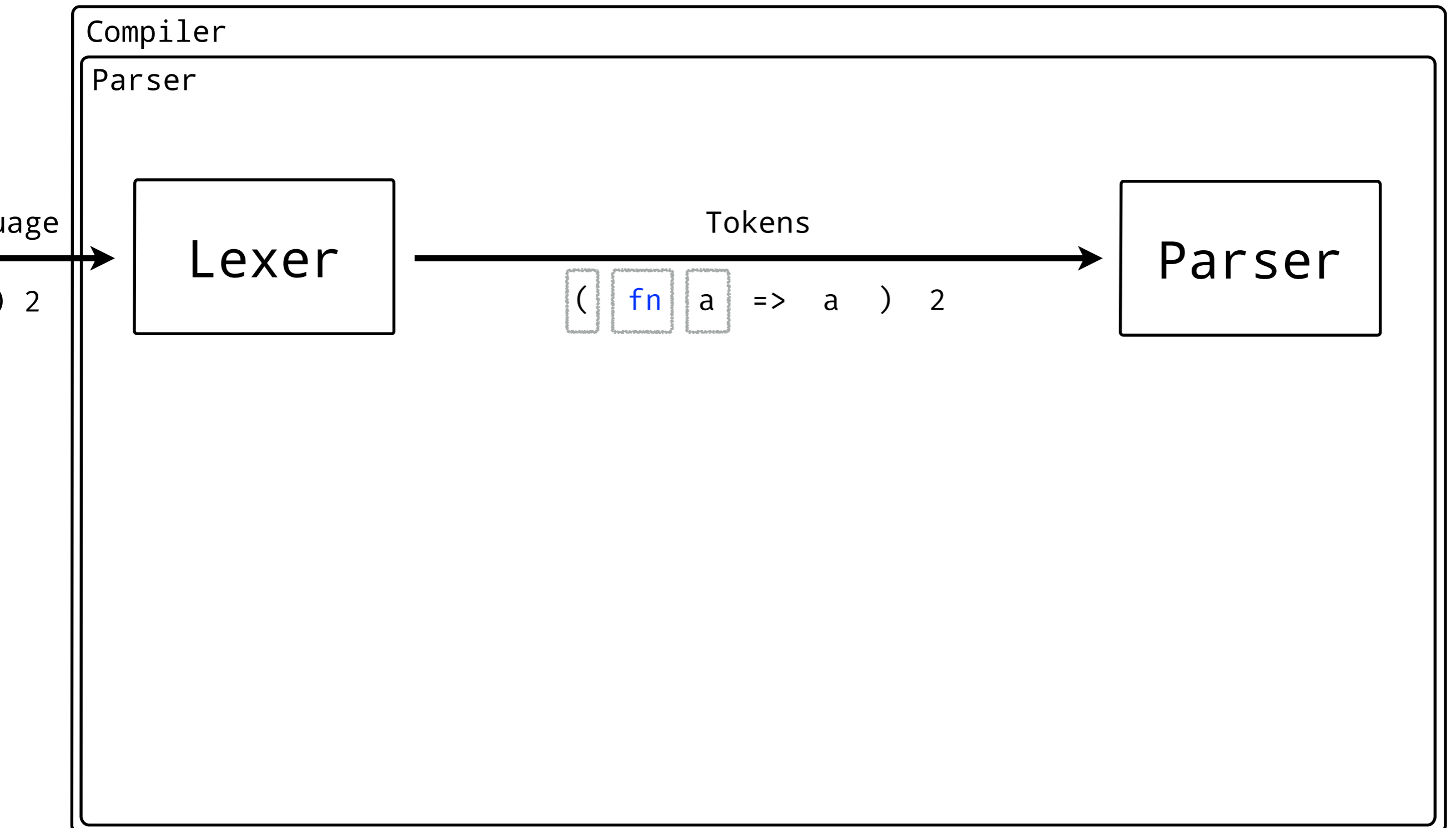
Lexing



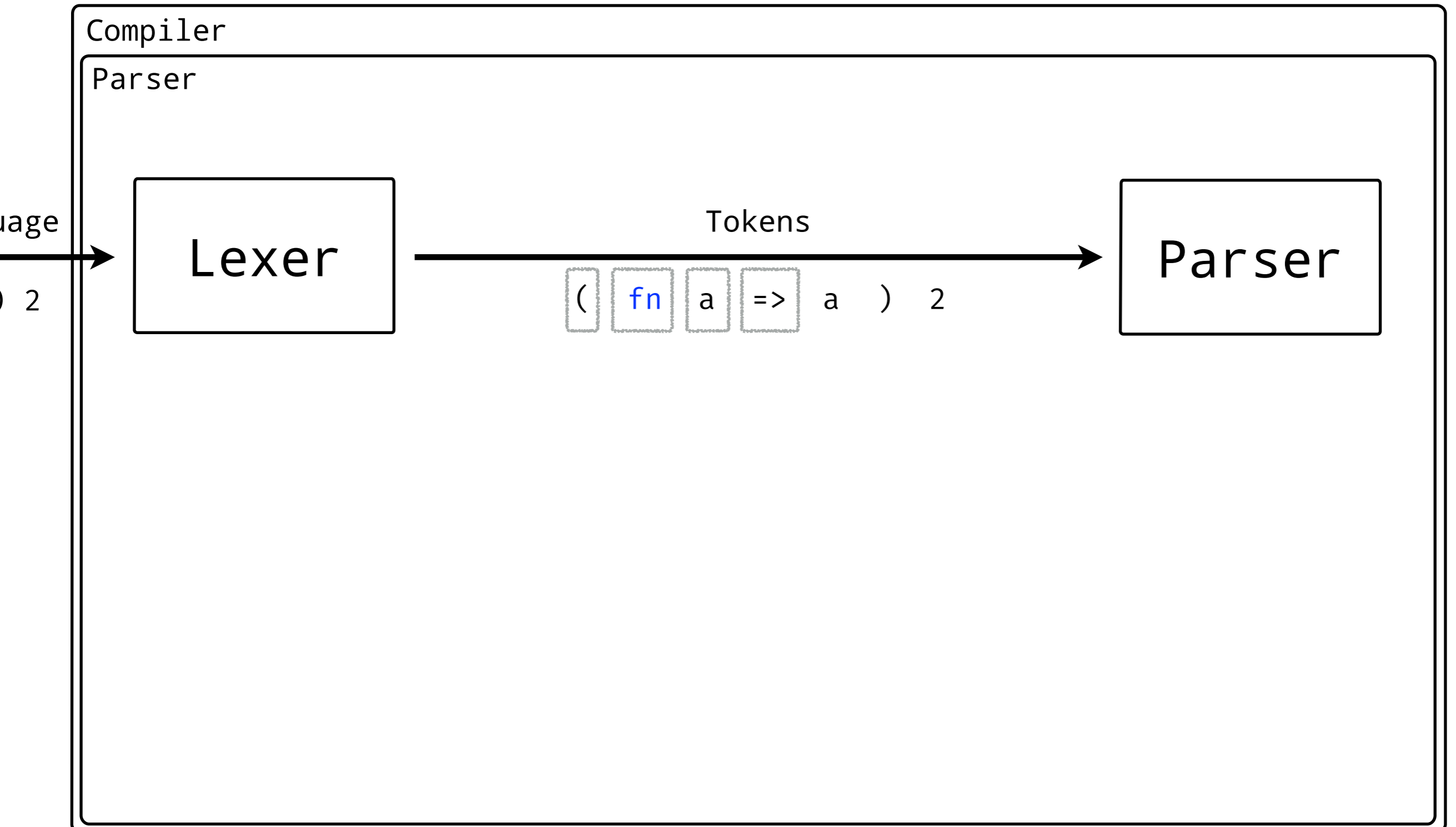
Lexing



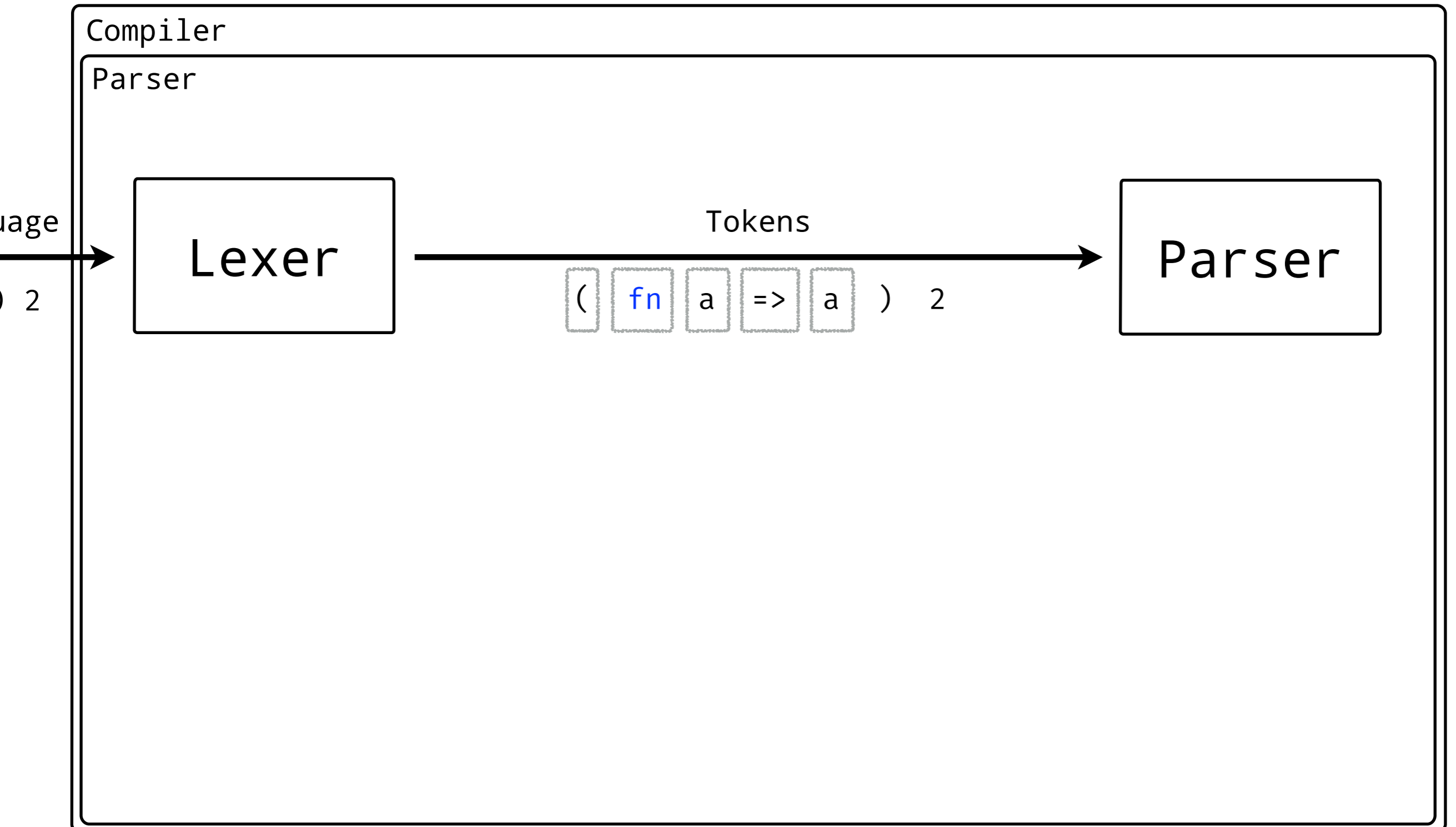
Lexing



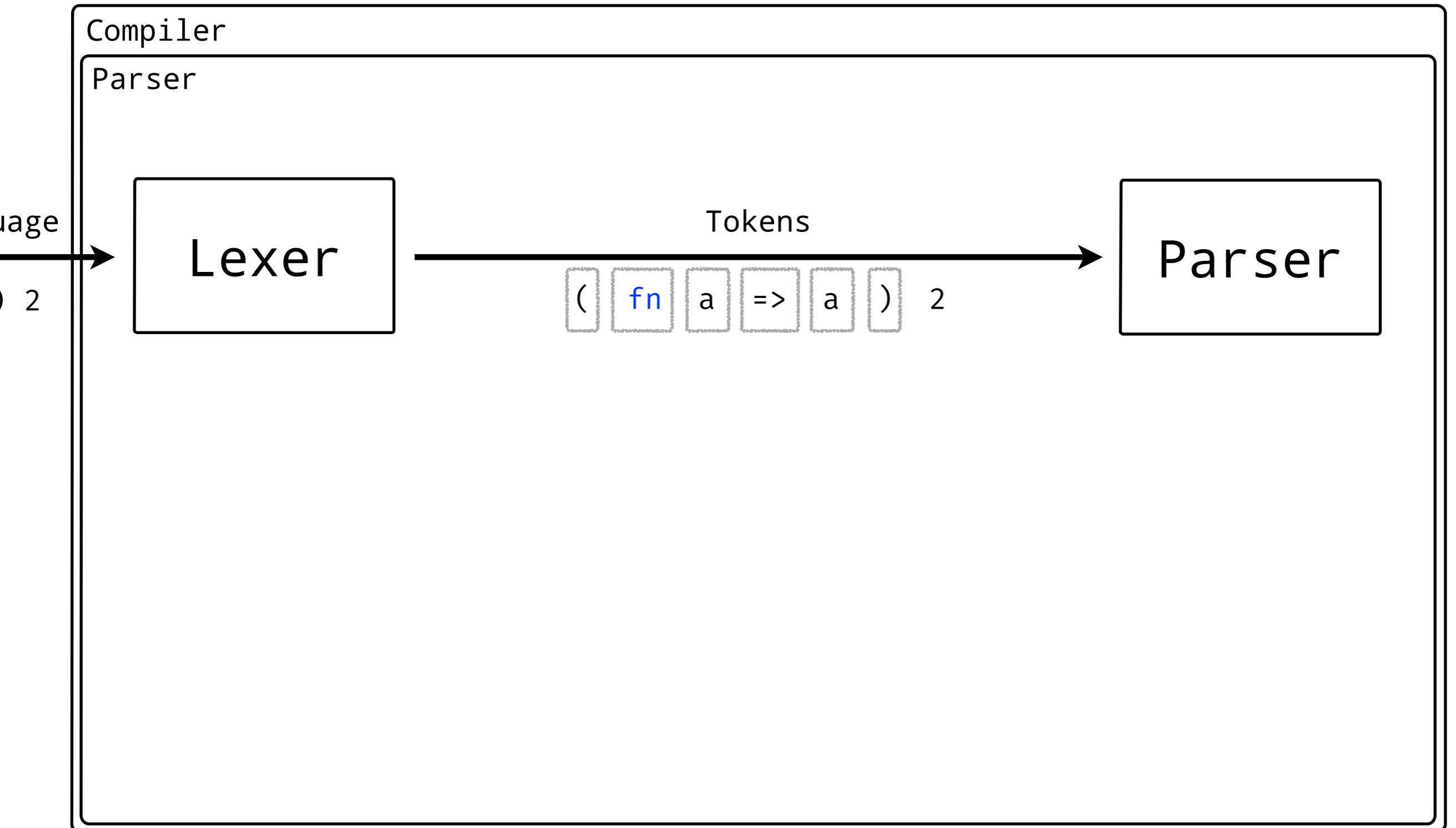
Lexing



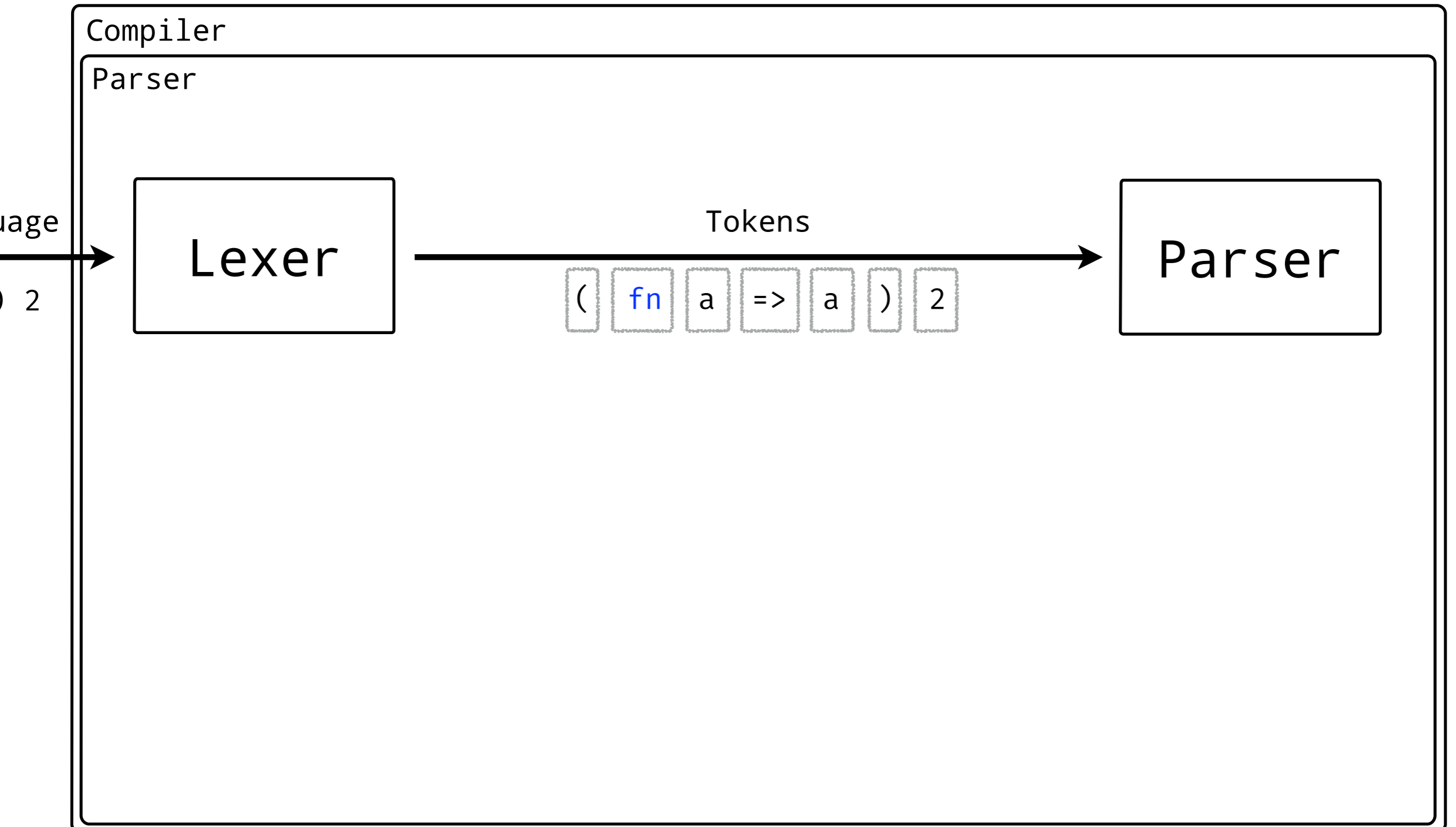
Lexing



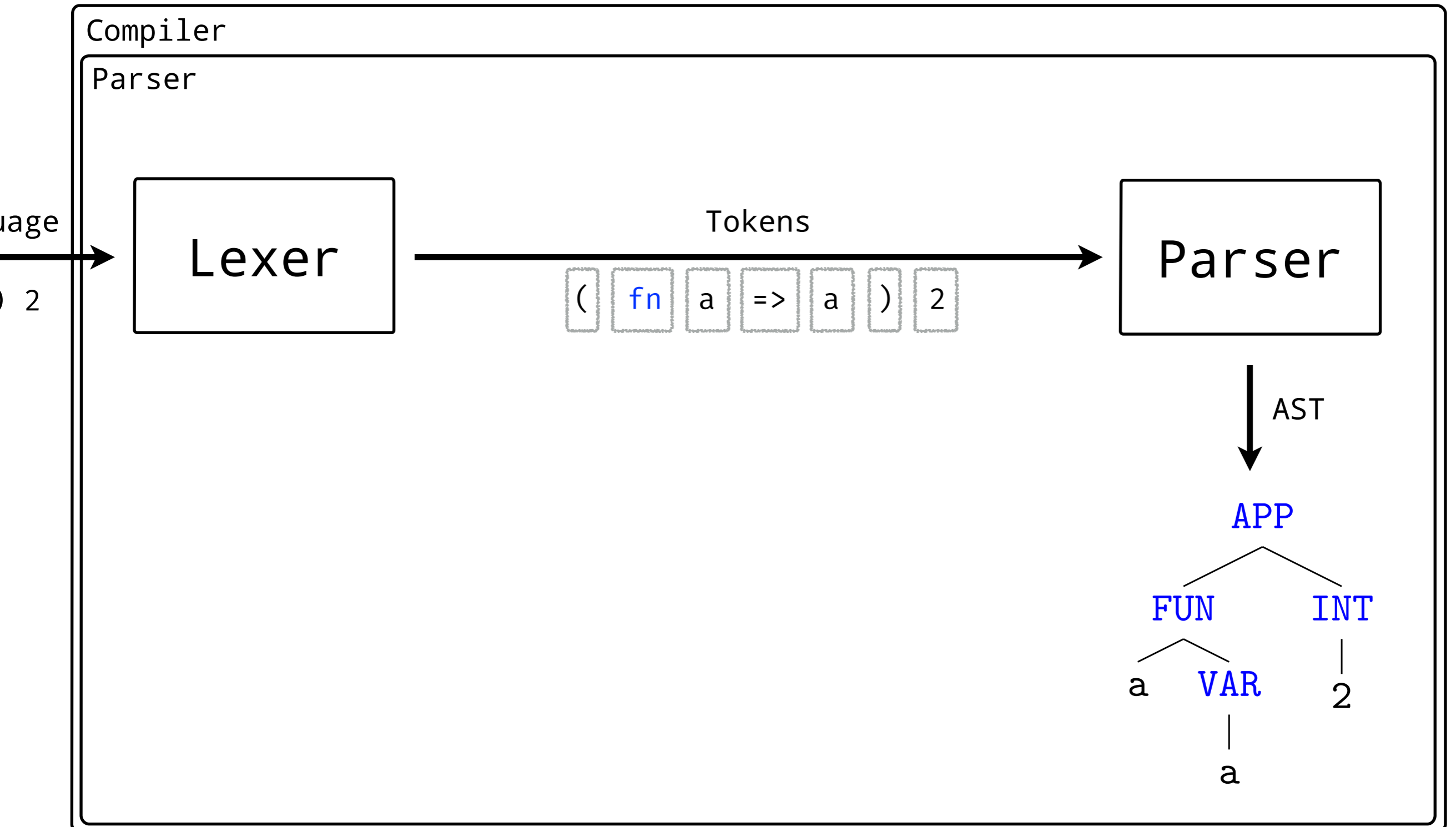
Lexing



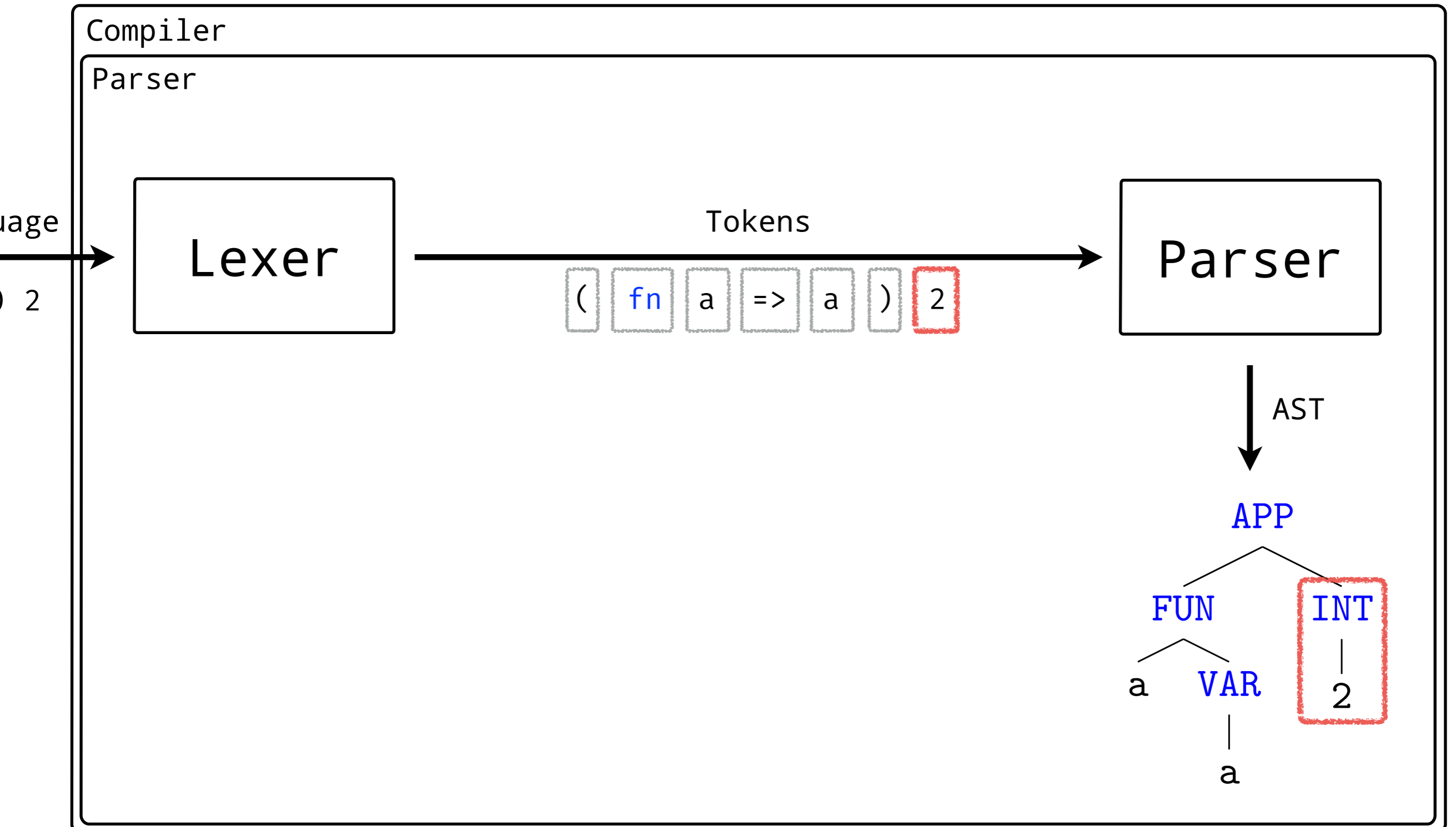
Lexing



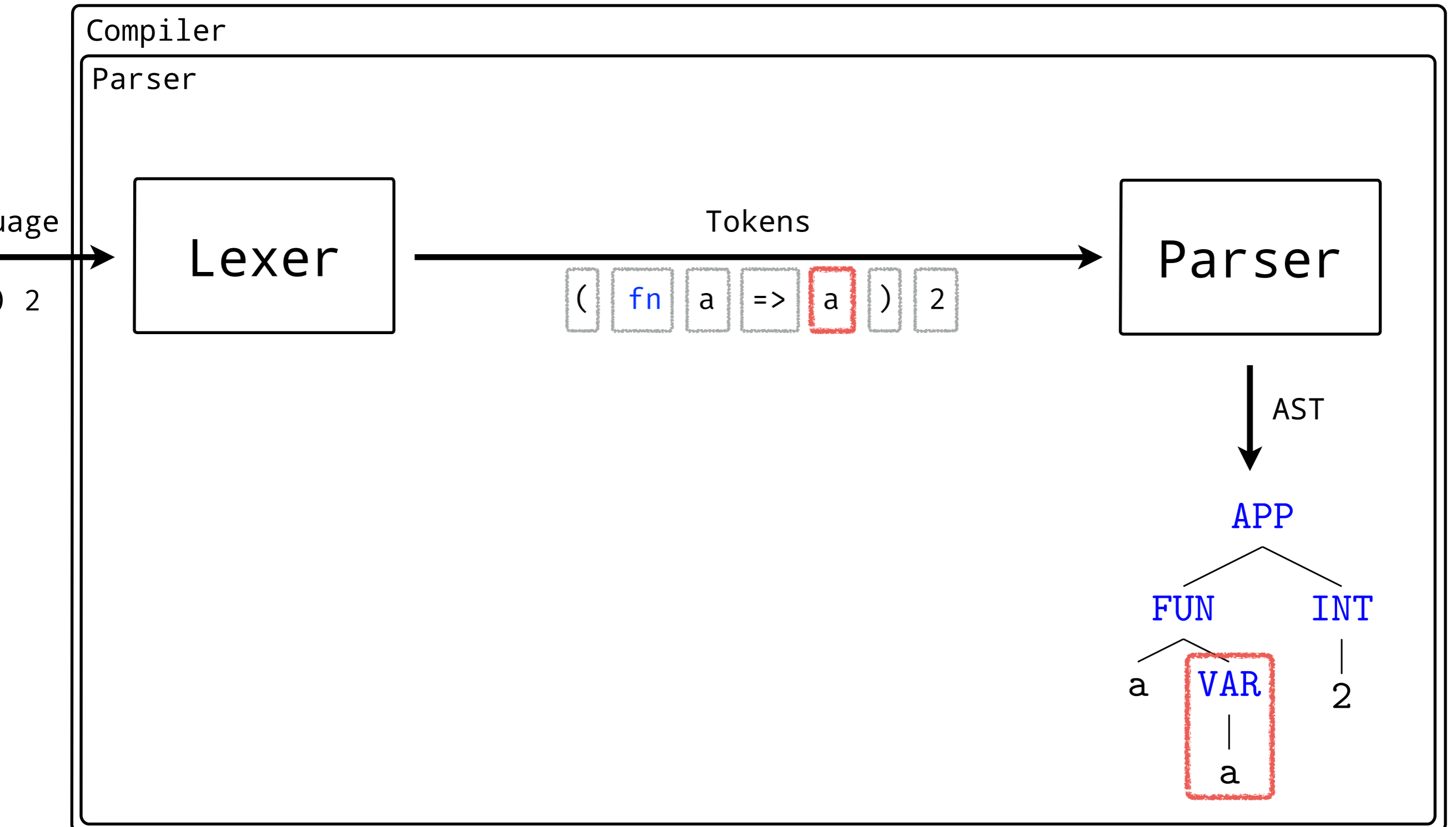
Lexing



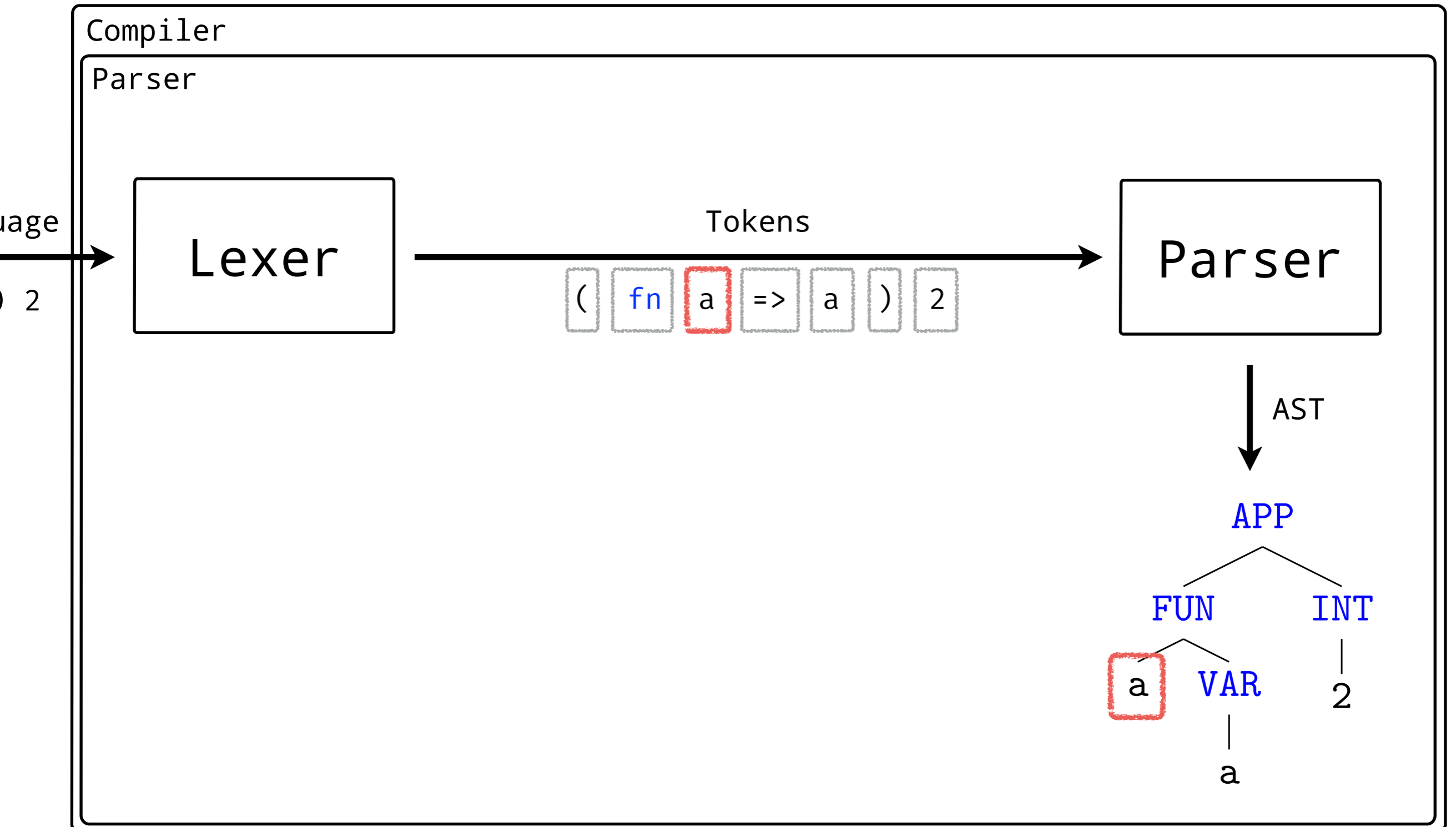
Parsing



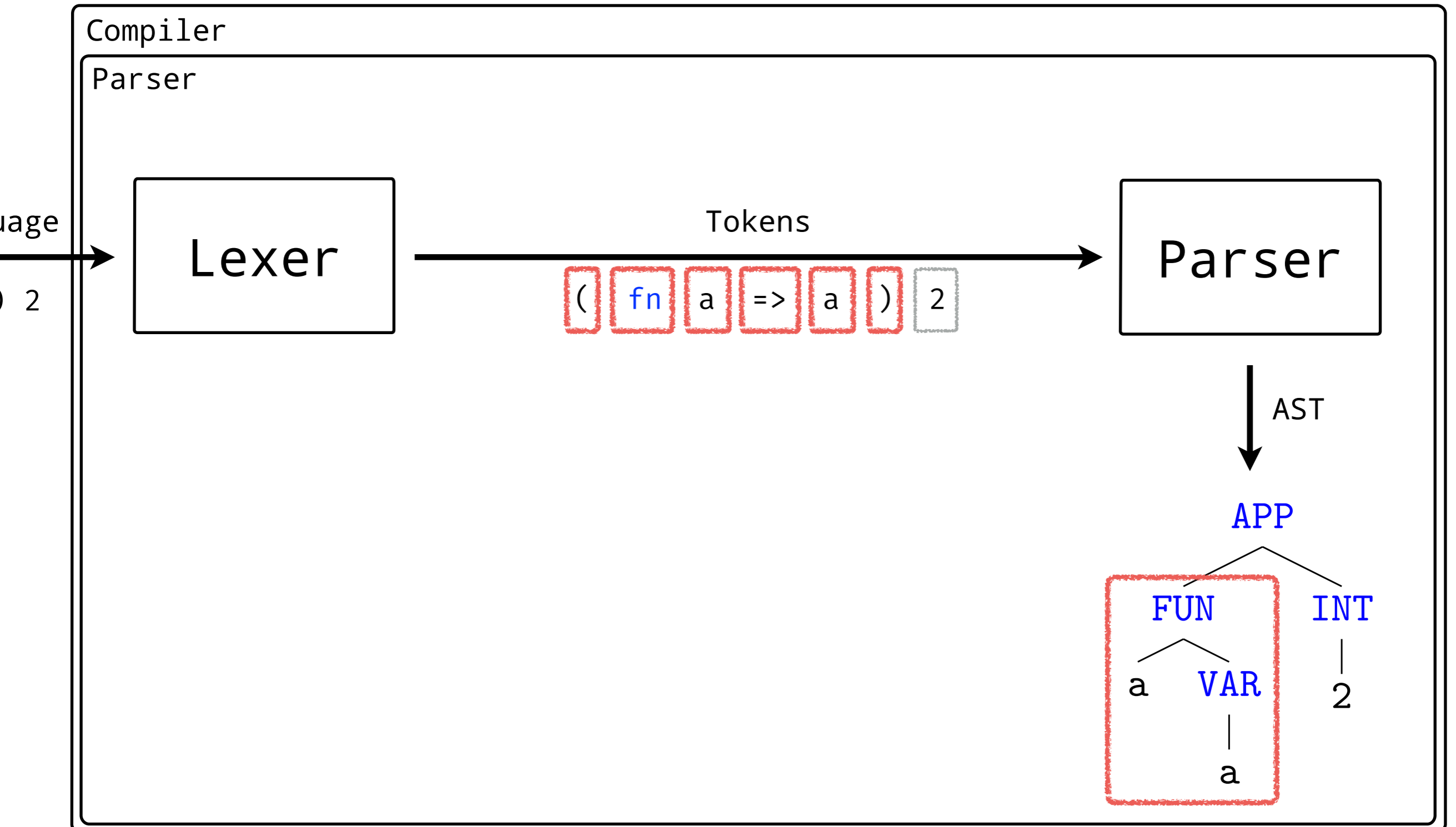
Parsing



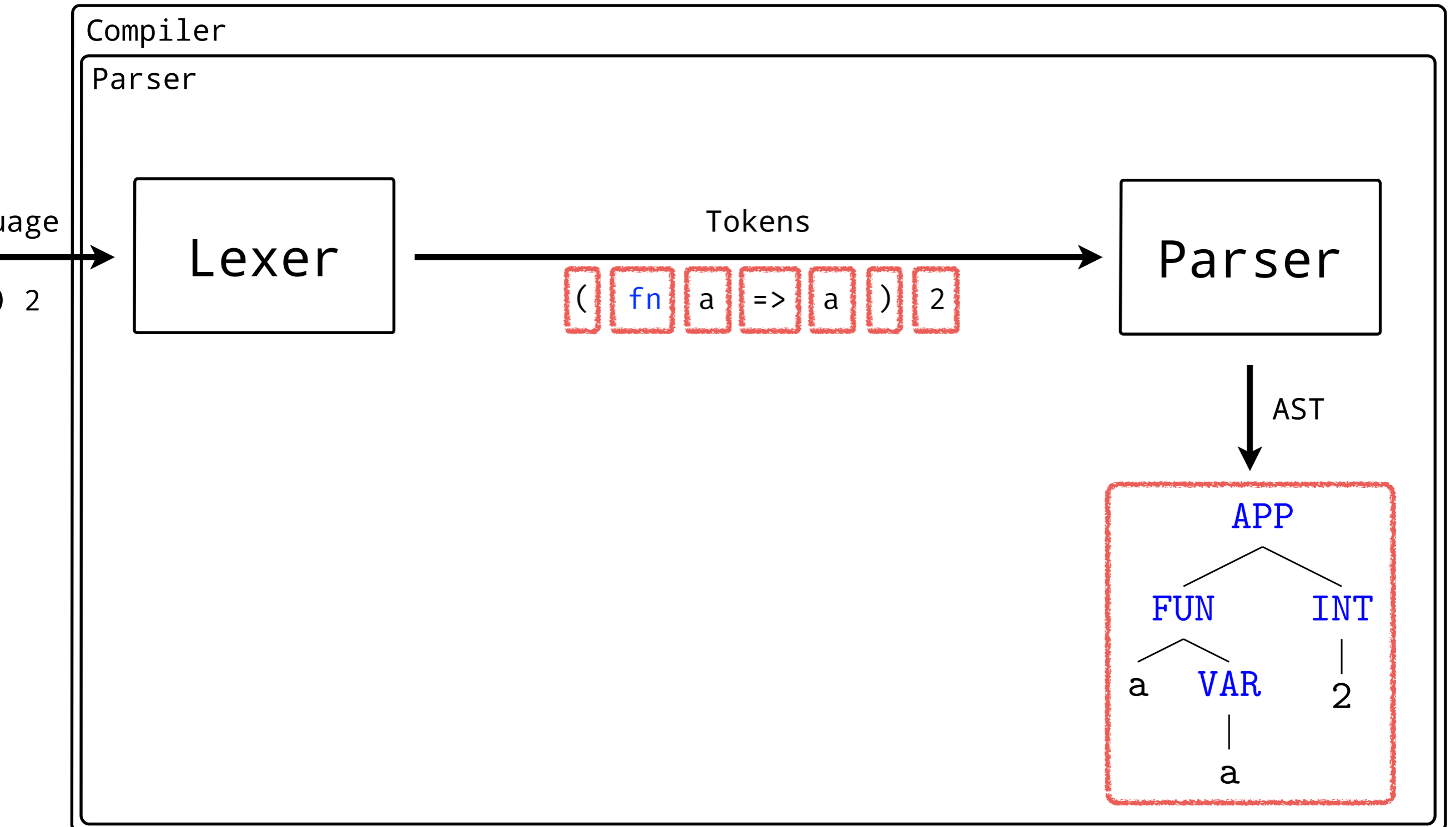
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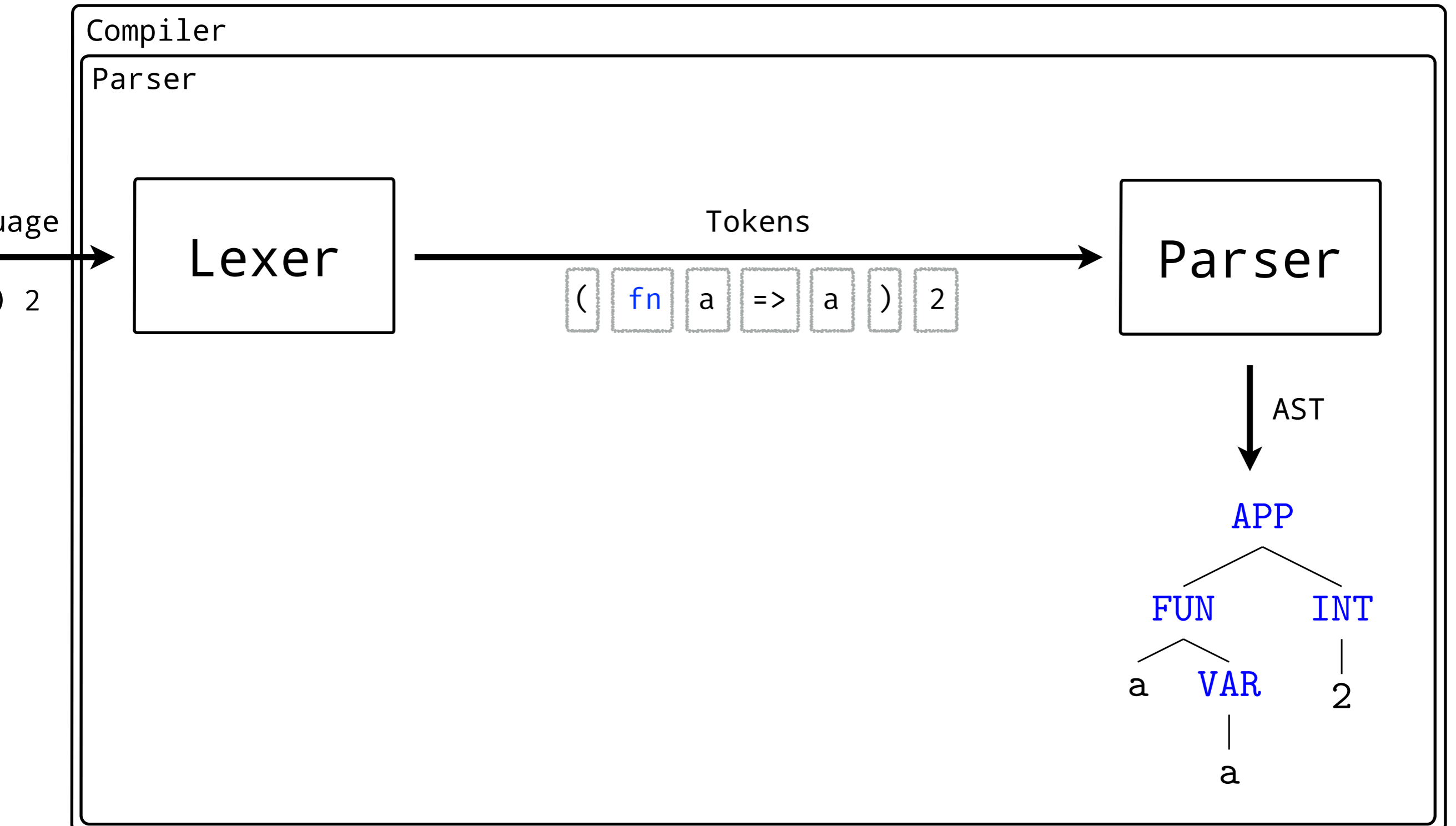
Parsing



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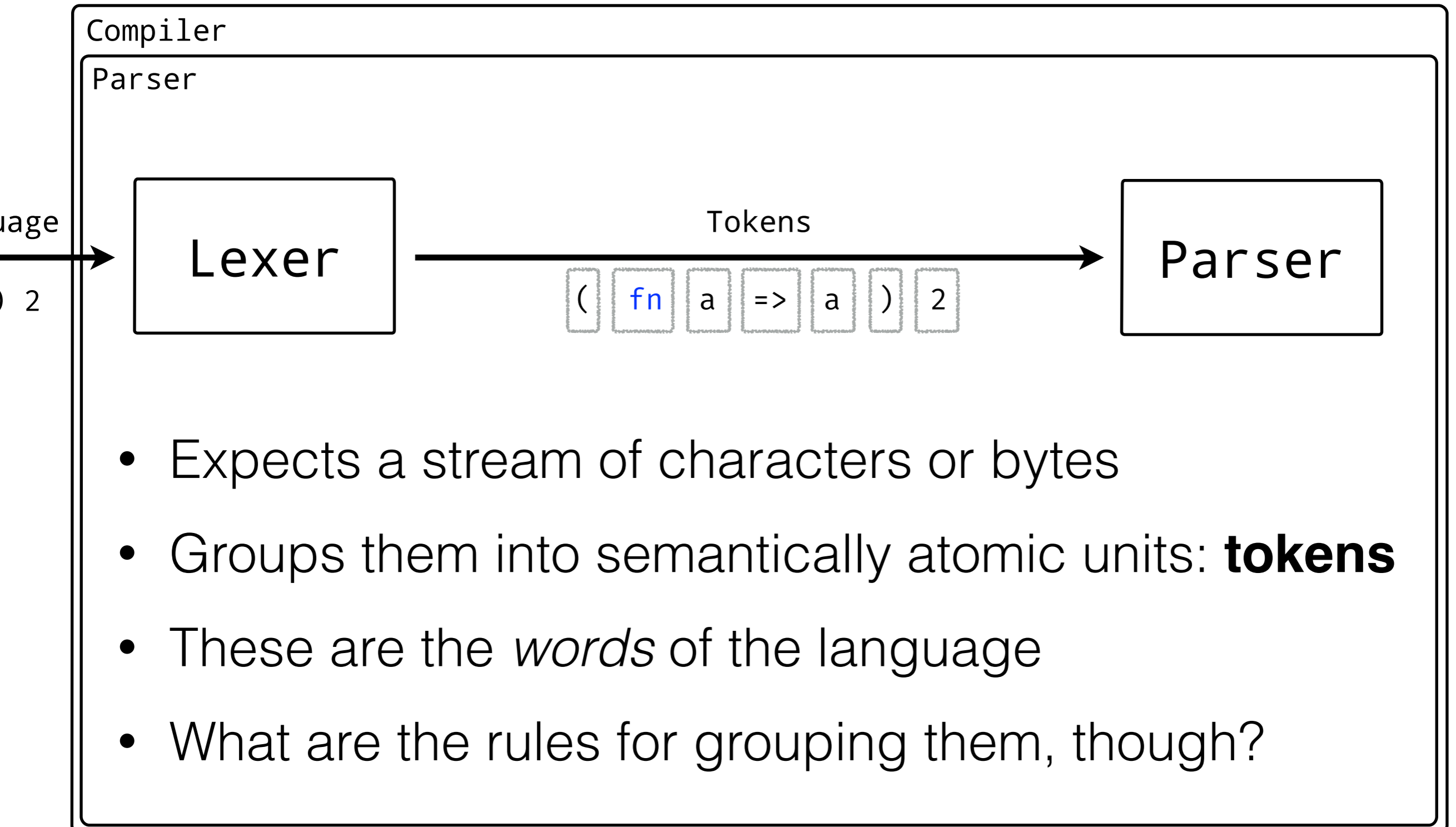


Lexing



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Lexing

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- Why not exactly that, though? Consider:

Lexing

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- Why not exactly that, though? Consider:

```
val sum = 1 + 2
```

```
val sum=1+2
```

```
val str = "spaces matter here"
```

Lexing

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 - By hand isn't hard either, just a little cumbersome
 - Lexer generators: Lex, Flex, Alex, ANTLR, etc.
- Lexing is what you need for syntax definition files

μML — Lexical Grammar

integers

identifiers

symbols

keywords

μML — Lexical Grammar

integers

`0|[1-9][0-9]*`

identifiers

symbols

keywords

μML — Lexical Grammar

integers

$0 | [1-9][0-9]^*$

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$[a-zA-Z]^+$

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$(,), +, -, =, =>$

keywords

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`(,), +, -, =, =>`

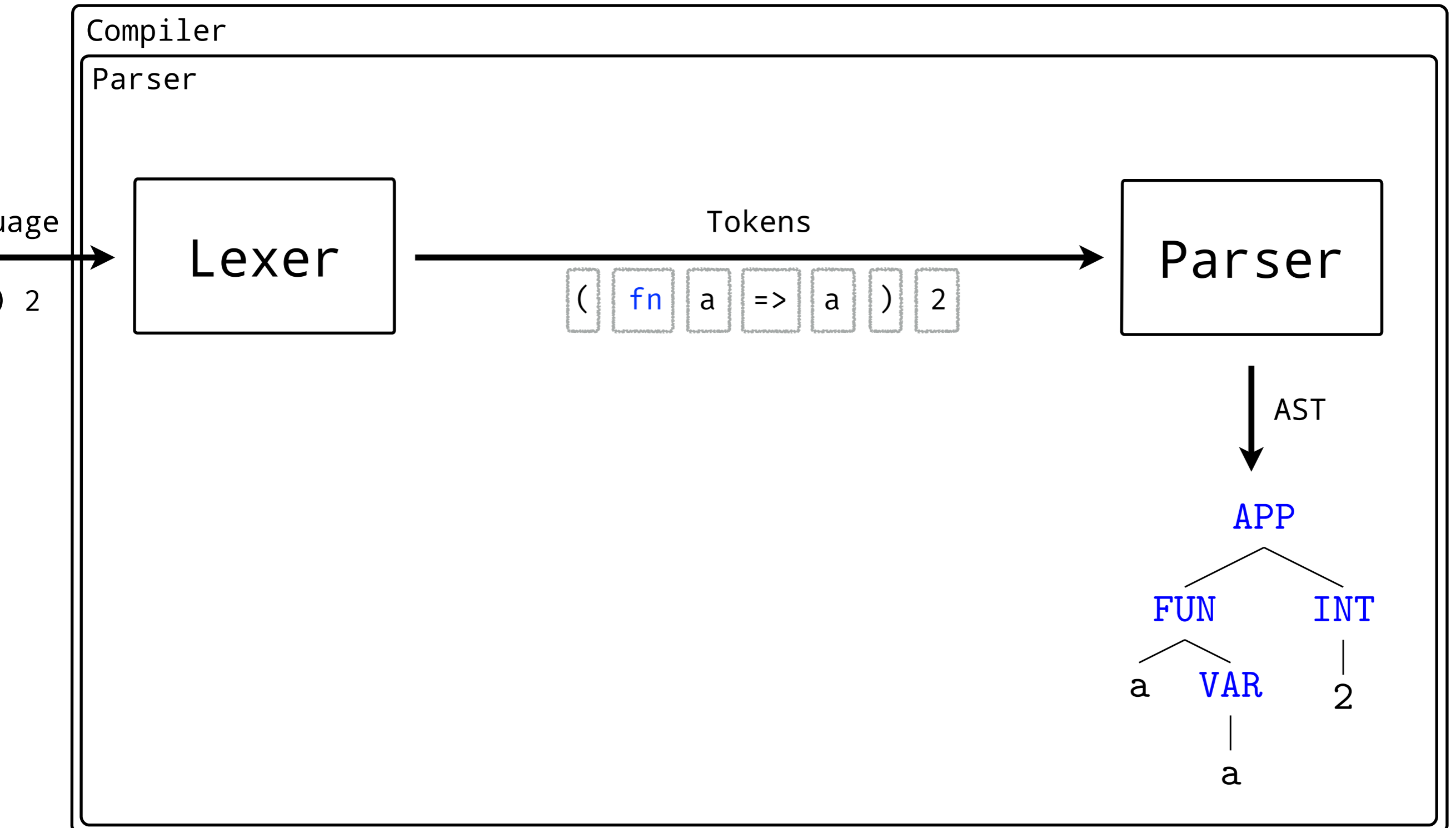
keywords

`if, then, else, let, val, in, end, fn, true, false`

Code

Parsing

Parsing



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Parsing

- The lexer recognizes valid words in the language
- Not all combinations of valid words form valid phrases in a language
- Syntactically correct: `val a = 1`
- Syntactically incorrect: `val val val`
- We must define the structure of phrases
- A syntactical grammar achieves that

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- Because they use a finite amount of memory
- Nesting needs a stack to remember the upper structures you're traversing
- Syntactical grammars express nesting using recursion



4427

You can't parse [X]HTML with regex. Because HTML can't be parsed by regex. Regex is not a tool that can be used to correctly parse HTML. As I have answered in HTML-and-regex questions here so many times before, the use of regex will not allow you to consume HTML. Regular expressions are a tool that is insufficiently sophisticated to understand the constructs employed by HTML. HTML is not a regular language and hence cannot be parsed by regular expressions. Regex queries are not equipped to break down HTML into its meaningful parts. so many times but it is not getting to me. Even enhanced irregular regular expressions as used by Perl are not up to the task of parsing HTML. You will never make me crack. HTML is a language of sufficient complexity that it cannot be parsed by regular expressions. Even Jon Skeet cannot parse HTML using regular expressions. Every time you attempt to parse HTML with regular expressions, the unholy child weeps the blood of virgins, and Russian hackers pwn your webapp. Parsing HTML with regex summons tainted souls into the realm of the living. HTML and regex go together like love, marriage, and ritual infanticide. The <center> cannot hold it is too late. The force of regex and HTML together in the same conceptual space will destroy your mind like so much watery putty. If you parse HTML with regex you are giving in to Them and their blasphemous ways which doom us all to inhuman toil for the One whose Name cannot be expressed in the Basic Multilingual Plane, he comes. HTML-plus-regex will liquify the nerves of the sentient whilst you observe, your psyche withering in the onslaught of horror. Regēx-based HTML parsers are the cancer that is killing StackOverflow *it is too late it is too late we cannot be saved* the transgression of a child ensures regex will consume all living tissue (except for HTML which it cannot, as previously prophesied) *dear lord help us how can anyone survive this scourge* using regex to parse HTML has doomed humanity to an eternity of dread torture and security holes *using regex as a tool to process HTML establishes a breach between this world and the dread realm of corrupt entities (like SGML entities, but more corrupt) a mere glimpse of the world of regex parsers for HTML will instantly transport a programmer's consciousness into a world of ceaseless screaming, he comes, the pestilent slithy regex-infection will devour your HTML parser, application and existence for all time like Visual Basic only worse he comes he comes do not fight he comes, his unholy radiance destroying all enlightenment, HTML tags leaking from your eyes like liquid pain, the song of regular expression parsing will extinguish the voices of mortal man from the sphere I can see it can you see if it is beautiful the final snuffing of the lies of Man ALL IS LOST ALL IS LOST the pony he comes he comes he comes the ichor permeates all MY FACE MY FACE on god no NO NOOOO NO stop the an*
 gles a r e not real ZALGO IS TONY THE PONY, HE COMES

Have you tried using an XML parser instead?



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It's not weird-looking Unicode characters that make regexes unsuitable for parsing.

... (more corrupt) a... instantly transport a programmer's... screaming, he comes, the pestilent slithy regex-infection... your HTML parser, application and existence for all time like Visual Basic only worse... he comes he comes do not fight he comes, his unholy radiance destroying all enlightenment, HTML tags leaking from your eyes like liquid pain, the song of regular expression parsing will extinguish the voices of mortal man from the sphere I can see it can you see if it is beautiful the final snuffing of the lies of Man ALL IS LOST ALL IS LOST the pony he comes he comes he comes the ichor permeates all MY FACE MY FACE on god no NO NOOOO NO stop the an... gles... e not real ZALGO IS TONY THE PONY, HE COMES

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Syntactical Grammar

μ ML — Syntactical Grammar

expr =

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`expr = int`

Here, blue symbols represent tokens coming from the lexer, not keywords.

μ ML — Syntactical Grammar

```
expr = int  
      | var
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μ ML — Syntactical Grammar

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expr = int  
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expr = int  
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      | bool  
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```

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μ ML — Syntactical Grammar

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expr = int  
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Introducing Precedence

- Function application has higher precedence over infix expressions in ML

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- `double 1 + 2 = (double 1) + 2`

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- $\text{double } 1 + 2 = (\text{double } 1) + 2$
- $\text{double } 1 + 2 \neq \text{double } (1 + 2)$

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- Grammars convey this by chaining rules in order of precedence
- Doesn't scale with many infix operators
- Use a special parser for that, e.g., the Shunting Yard algorithm

Introducing Precedence

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      | bool  
      | ( expr )  
      | fn var => expr  
      | if expr then expr else expr  
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      | expr expr
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```
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```

Introducing Precedence

expr =

bool = true | false

oper = + | -

```
atomic = int
        | var
        | bool
        | ( expr )
        | let val var = expr in expr end
```

Introducing Precedence

```
expr =
```

```
bool = true | false
```

```
oper = + | -
```

```
app = atomic  
    | app atomic
```

```
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        | bool  
        | ( expr )  
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```

Introducing Precedence

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expr =
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```
oper = + | -
```

```
infix = app  
      | infix oper infix
```

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     | app atomic
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```

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Parsing Strategies

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- Parser generators: YACC, ANTLR, Bison, etc.

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- **Parser combinators** are an abstraction over this idea

Code

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- The problem is that some rules are left-recursive, i.e., the rule itself appears as the first symbol on the left
- This is problematic for a recursive descent parser because the structure of function calls follow the structure of rule definitions
- That means infinite recursion in the parser, which isn't good

Left-Recursive Grammar

```
expr = infix  
      | fn var => expr  
      | if expr then expr else expr
```

```
bool = true | false  
oper = + | -
```

```
infix = app  
       | infix oper infix
```

```
app = atomic  
     | app atomic
```

```
atomic = int  
        | var  
        | bool  
        | ( expr )  
        | let val var = expr in expr end
```

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```
app = atomic  
    | atomic atomic  
    | atomic atomic atomic  
    | atomic atomic atomic atomic  
    ...
```


Left-Recursive Grammar

```
app = atomic  
    | atomic atomic  
    | atomic (atomic atomic)  
    | atomic (atomic (atomic atomic))  
    ...
```

Left-Recursive Grammar

```
app = atomic { app }
```

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      | fn var => expr  
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      | app oper app oper infix
      | app oper app oper app oper infix
      ...
```


Removing Left-Recursion

```
infix = app
      | app (oper infix)
      | app (oper app (oper infix))
      | app (oper app (oper app (oper infix)))
      ...
```

Removing Left-Recursion

```
infix = app { oper infix }
```

Removing Left-Recursion

```
expr = infix  
    | fn var => expr  
    | if expr then expr else expr
```

```
bool = true | false  
oper = + | -
```

```
infix = app { oper infix }
```

```
app = atomic { app }
```

```
12 14 13  
(12 14) 13
```

```
atomic = int  
        | var  
        | bool  
        | ( expr )  
        | let val var = expr in expr end
```

[github.com / igstan / itake-2016](https://github.com/igstan/itake-2016)

Homework

- Write a lexer for JSON
- Write a recursive descent parser for JSON
- It's way easier than today's vehicle language
- I promise!
- Specification: json.org

Thank You!

Questions!